NONSYMMETRICAL FLOWS OF A VISCOUS LIQUID COOLING UNDER SYMMETRICAL CONDITIONS

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Apparatus in which a liquid is cooled passing through a bundle of "parallel" tubes (fed by a common source) is used actively in industry. This includes tubular heat exchangers, systems for pumping heated oil along parallel pipelines, etc. Operating practice of them indicates that in some cases some of the tubes do not operate in the required production regime, although all of the tubes are under identical conditions. In this work a study is made of disturbance of symmetry for flow of a liquid being cooled through a pair of identical parallel tubes.

It was demonstrated in [1] that the head-discharge characteristic (HDC) of flow of cooling viscous liquid contains a descending section within whose limits instability is possible. In [2], where heating was not supplied to the input, but occurred as a result of dissipation, a similar HDC was obtained, and a study was made of jumps in delivery with a smooth change of pressure in the vicinity of the limits of the descending section. However, with recording of delivery for the descending section of the HDC from the point of view of a point system considered in [2], it is stable. Statement of the problem for two tubes [3] makes it possible to consider possible disturbance of flow symmetry. In the case of a liquid being cooled, solution of this problem indicates that nonsymmetrical flows form with the existence of lower values of pressure than in [2, 3].

1. We consider cooling by a viscous incompressible liquid with flow through two parallel joined tubes with a prescribed common delivery. With a series of simplifying assumptions [3] flow is described by the following set of equations in dimensionless variables:

$$\partial\Theta_1/\partial\tau + \omega_1\partial\Theta_1/\partial\xi = -\Theta_1 - \beta(\Theta_1 - \Theta_2); \qquad (1.1)$$

$$\partial\Theta_2/\partial\tau + \omega_2\partial\Theta_2/\partial\xi = -\Theta_2 + \beta(\Theta_1 - \Theta_2); \qquad (1.2)$$

$$\omega_1\int_0^1 \exp(-\Theta_1)\,d\xi = \omega_2\int_0^1 \exp(-\Theta_2)\,d\xi = \Pi; \qquad (1.3)$$

$$\omega = \omega_1 + \omega_2, \qquad (1.4)$$

where $\Theta = (T - T_0)U/RT_0^2$; $\omega = c\rho Q/(2\alpha\ell\pi r)$; $\tau = 2\alpha t/(c\rho r)$; $\xi = z/\ell$; $\beta = \alpha_1/\alpha$; $\Pi = \Delta p c\rho r^3/(16\mu(T_0)\alpha\ell^2)$; $\mu = \mu_0\exp(U/RT)$; T is liquid temperature; T_0, temperature of the surroundings; Q, delivery; c, heat capacity; ρ, liquid density; α, heat-transfer coefficient; r, radius; ℓ, tube length; z, coordinate along the tube; t, time; α_1, heat-transfer coefficient between tubes; Δp, pressure drop in the tubes between the inlet and outlet; μ, liquid dynamic viscosity; indices 1 and 2 relate to the first and second tubes. In recording the temperature dependence for viscosity in Eq. (1.3) use is made of the Frank–Kamenetskii transform [4].

In contrast to [3], dissipative heat release is assumed to be negligibly small and, therefore, the corresponding terms in the equations are omitted. The effects in question are determined by the condition at the inlet

$$\xi = 0:\quad \Theta_1 = \Theta_2 = \Theta_0,\quad \Theta_0 = (T^0 - T_0)U/RT_0^2 > 0 \qquad (1.5)$$

(T^0 is liquid temperature at the inlet). In addition, in (1.1) and (1.2) heat exchange between tubes is considered. Known parameters are assumed to be total liquid delivery ω, thermal head at the inlet to the tubes Θ_0, and heat-exchange coefficient between the tubes β. Steady temperature profiles, liquid delivery in the tubes Θ_1, Θ_2, ω_1, ω_2, and pressure drop Π are determined.

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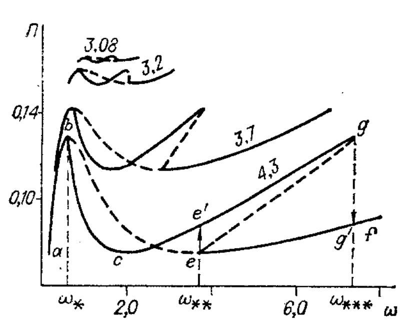

Fig. 1

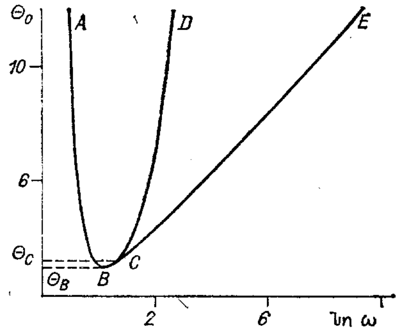

Fig. 2

2. First let heat exchange between the tubes be absent ($\beta = 0$). Then according to (1.1), (1.2), and (1.5), the steady profile $\Theta_1(\xi)$ depends only on ω_1, and $\Theta_2(\xi)$ depends only on ω_2. Whence it follows that each of the tubes has the HDC found in [1]. The characteristic for the system of two tubes is obtained by adding their HDC. In fact, since pressure drop Π in both tubes is the same, delivery $\omega(\Pi) = \omega_1(\Pi) + \omega_2(\Pi)$. As indicated in [1], with $\Theta_0 > \Theta_{min}$ the HDC (for one tube) is N-shaped, and in region $\Pi_d < \Pi < \Pi_u$ each value of Π relates to three values of ω (slow, intermediate, high). Combination of them gives six branches for $\omega(\Pi)$. As a result of this, the HDC for the system of two tubes in question are obtained and are shown in Fig. 1 (alongside curves for values of Θ_0); N-shaped curve abef, similar to the HDC for one tube, corresponds to symmetrical flow with uniform delivery and temperature profiles in both tubes, branch bcge corresponds to nonsymmetrical flows, section bc corresponds to the case when in one tube flow is realized with low delivery, and in the other tube with intermediate delivery. Section cg relates to flow with low delivery in one tube and high delivery in the other, ge relates to flow with a combination of intermediate and high delivery values in the tubes. A study of the stability of steady-state solutions with small perturbations (see below) indicates that flows corresponding to HDC sections be and ge are unstable (broken curves in Fig. 1).

With $\omega < \omega_*$ total delivery is distributed equally between the tubes, liquid cooling in the tubes proceeds in the same way, and symmetrical flow is realized. On reaching ω_* this flow becomes unstable, development of nonsymmetric flow with $\omega > \omega_*$ proceeds in a soft regime, and the pressure drop decreases (section bc in Fig.1). As ω increases a greater part of the liquid will flow through one tube, and this means that the liquid temperature in this tube will be higher than in the other. Nonsymmetrical flow remains until $\omega < \omega_{***}$, and in this way in section cg the pressure drop increases. With $\omega = \omega_{***}$ there is restoration of disturbed symmetry (in a hard regime), and the pressure drop falls jumpwise ($g \to g'$ in Fig. 1). With a further increase in total delivery flow remains symmetrical. If now ω decreases, then disturbance of flow symmetry proceeds with $\omega = \omega_{**}$ in a hard regime, and restoration proceeds with $\omega = \omega_*$ in a soft regime. If Θ_0 decreases, values of ω_{**} and ω_{***} approach each other, then a hard regime changes into a soft regime and, finally, the region of nonsymmetrical flow disappears.

Shown in Fig. 2 is the region for existence of nonsymmetrical flows, and it lies beneath curve ABCE. Curve ABCD corresponds to the boundary of instability for symmetrical flow, so that DCE is the boundary of the region for bistability. If ω increases from low values, then disturbance of flow symmetry in a soft regime proceeds in line AB, symmetrical flow again becomes stable in line BCD, and restoration of flow symmetry in a hard regime proceeds in line CE. There is a region of values $\Theta_B < \Theta_0 < \Theta_C$ in which disturbance and restoration of flow symmetry proceeds in a soft regime (see the HDC in Fig. 1 with $\Theta_0 = 3.08$). With $\Theta_0 < \Theta_B$ all of the critical phenomena degenerate, and in the HDC sections with a negative flow are absent.

For flow of ethylene glycol heated initially to $T^0 = 120$°C through a tube 1 m long and 1 cm in diameter at ambient temperature $T_0 = 20$°C calculation using values of ρ, c, μ from [5] and heat-transfer coefficient $\alpha = 10$ W/(m^2·K) gives a delivery value with which symmetry is disturbed, $Q_* = 0.13$ cm^3/sec. In this way head $\Delta p = 1.96$ Pa. Symmetry is restored with $Q_{***} = 0.69$ cm^3/sec.

From the method of plotting the HDC for two tubes it follows that in those cases when the HDC is nonmonotonic (nonmonotonicity may be due to different reasons) the effect of flow symmetry disturbance might also be expected in a system of n parallel tubes.

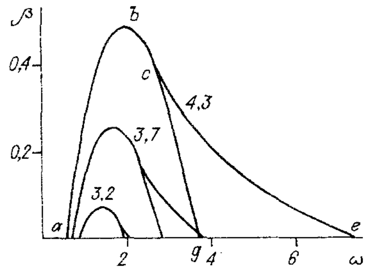

Fig. 3

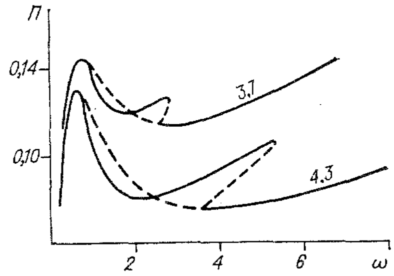

Fig. 4

3. With $\beta \neq 0$ steady-state values of delivery with nonsymmetrical flow are not determined from the HDC for one tube. In addition, the branch for symmetrical flow in the HDC ($a$bef in Fig. 1) does not change, since with $\Theta_1 = \Theta_2$ there is no heat exchange between the tubes. After substituting $\omega_1 = \omega/2 + v$, $\omega_2 = \omega/2 - v$, equality (1.4) is satisfied homologously, and steady-state solution (1.1), (1.2) with boundary condition (1.5) takes the form

$$\Theta_1 = [(\omega\beta + q + 2v)\exp(k_1\xi) - (\omega\beta - q + 2v)\exp(k_2\xi)]\Theta_0/(2q); \tag{3.1}$$

$$\Theta_2 = [(\omega\beta + q - 2v)\exp(k_1\xi) - (\omega\beta - q - 2v)\exp(k_2\xi)]\Theta_0/(2q), \tag{3.2}$$

where $q = \sqrt{\beta^2\omega^2 + 4v^2(1+2\beta)}$; $k_1 = 2\dfrac{q-(1+\beta)\omega}{\omega^2-4v^2}$; $k_2 = 2\dfrac{-q-(1+\beta)\omega}{\omega^2-4v^2}$.

Equation (1.3) is rewritten as

$$F(v) = 0, \quad F(v) = (\omega/2 + v)\int_0^1 \exp(-\Theta_1)\,d\xi - (\omega/2 - v)\int_0^1 \exp(-\Theta_2)\,d\xi. \tag{3.3}$$

Substitution of (3.1) and (3.2) in (3.3) gives an equation for determining v which with all values of parameters has the solution v = 0, which relates to symmetrical flow. Appearance of solutions $v \neq 0$ (corresponding to nonsymmetrical flows) relates to values of parameters Θ_0, ω, and β which satisfy the equation $F_v(0) = 0$ (the index represents differentiation with respect to v) having the form

$$2\int_0^1 \left[1 + \exp\left(-\frac{4\beta}{\omega}\xi\right)\right]\exp\left[-\Theta_0\exp\left(-\frac{2\xi}{\omega}\right)\right]d\xi = \frac{\omega}{2\beta}\left[1 - \exp\left(-\frac{4\beta}{\omega}\right)\right]\exp\left[-\Theta_0\exp\left(-\frac{2}{\omega}\right)\right]. \tag{3.4}$$

With $\beta = 0$ in Fig. 2, Eq. (3.4) relates to curve ABCD. With $\beta \neq 0$ the qualitative form of the curve in plane (ω, Θ_0) is retained, and with an increase in β the curves shift upward. Values of parameters relating to disappearance of nonsymmetrical flow ($v \neq 0$) are determined from the condition

$$F_v(v) = 0, \tag{3.5}$$

considered together with (3.3). With $\beta = 0$ the solution is shown in Fig. 2 (curve CE). By means of (3.3)-(3.5) the plane of parameters (ω, β) is broken down into regions of realizing symmetrical and nonsymmetrical flows (Fig. 3). Boundaries of this region are plotted with different values of Θ_0 (shown on the lines). Within region $a$bg only nonsymmetrical flows are possible; outside region $a$be only symmetrical flows are possible, and gce bounds the region of bistability. It can be seen that an increase in β (as also a reduction in Θ_0) reduces the region for nonsymmetrical flows.

Shown in Fig. 4 is the HDC with $\beta = 0.1$ for two values of Θ_0. In contrast to the solution with $\beta = 0$ (see Fig. 1), the point for loss of stability of symmetrical flow does not coincide with the extreme of the characteristic.

4. Stability of steady-state solutions has been studied by means of a linearized set of equations for small deviations $u_n(\xi)\exp(\lambda\tau)$ and $w\exp(\lambda\tau)$ from steady-state solutions $\Theta_{ns}(\xi)$, v_s:

$$(\omega/2 + v_s)dz_1/d\xi = -(1 + \beta + \lambda)z_1 + \beta z_2 - d\Theta_{1s}/d\xi; \tag{4.1}$$

$$(\omega/2 - v_s)dz_2/d\xi = -(1 + \beta + \lambda)z_2 + \beta z_1 + d\Theta_{2s}/d\xi; \qquad (4.2)$$

$$\int_0^1 [1 - (\omega/2 + v_s) z_1] \exp(-\Theta_{1s})\, d\xi + \int_0^1 [1 + (\omega/2 - v_s) z_2] \exp(-\Theta_{2s})\, d\xi = 0, \qquad (4.3)$$

$$z_n(0) = 0,$$

where $z_n = u_n/w$, $n = 1, 2$.

Substitution of solutions (4.1) and (4.2) in (4.3) leads to the equation $f(\lambda) = 0$ for determining the natural values of λ. The steady-state solution is stable if function $f(\lambda)$ of complex variable λ does not revert to zero in the right-hand half-plane $\mathrm{Re}\lambda > 0$. The number of zeros for function $f(\lambda)$ in the right-hand half-plane may be determined by means of the argument principle [6]

$$N - P = \frac{1}{2\pi} \Delta_C \arg f(\lambda), \qquad (4.4)$$

where N is number of zeros; P is number of poles for $f(\lambda)$ in the right-hand half-plane, and the right-hand part means the increment in the argument of function $f(\lambda)$ with a circuit of contour C embracing the right-hand part of the half-plane divided by 2π. It is convenient to select a semicircle of infinite radius as contour C.

Function $f(\lambda)$ with $\beta \neq 0$ is quite cumbersome and, therefore, below for simplicity equations are provided with $\beta = 0$, and then differences will be indicated occurring with $\beta \neq 0$. With $\beta = 0$,

$$f(\lambda) = \sum_{n=1}^{2} \int_0^1 \left\{1 - \Theta_{ns}\lambda^{-1}\left[1 - \exp\left(-\frac{\lambda}{\omega_{ns}}\xi\right)\right]\right\} \exp(-\Theta_{ns})\, d\xi. \qquad (4.5)$$

In the right-hand half-plane this function has no poles. In order to determine the right-hand part in (4.4) consideration should be given to the corresponding contour in complex plane f which is a reflection of contour C. The whole infinite semicircle ($\lambda \to \infty$) turns into a point at the real axis lying to the right of zero $\left(\sum_{n=1}^{2} \int_0^1 \exp(-\Theta_{ns}) d\xi > 0\right)$. The imaginary axis ($\lambda = i\nu$) is reflected in a curve

$$f(i\nu) = \sum_{n=1}^{2} \left\{ \int_0^1 \left[1 - \Theta_{ns}\nu^{-1} \sin\left(\frac{\nu}{\omega_{ns}}\xi\right)\right] \exp(-\Theta_{ns})\, d\xi + i \int_0^1 \Theta_{ns}\nu^{-1} \times \right.$$
$$\left. \times \left[1 - \cos\left(\frac{\nu}{\omega_{ns}}\xi\right)\right] \exp(-\Theta_{ns})\, d\xi \right\}.$$

It can be seen that the sign of the imaginary part of $f(i\nu)$ is governed by the sign of ν. Whence it follows that the positive (negative) imaginary axis is reflected in a curve lying above (beneath) the real axis of plane f. Reflection of the zero point for plane λ lies on the real axis to the left of point $f(\infty)$:

$$f(0) = \sum_{n=1}^{2} \int_0^1 (1 - \xi\omega_{ns}^{-1}\Theta_{ns}) \exp(-\Theta_{ns})\, d\xi = \sum_{n=1}^{2} d\Pi/d\omega_{ns}. \qquad (4.6)$$

Two cases are possible.

1) The zero point for plane λ is reflected at a point lying on the real axis to the right of the zero point [$f(0) > 0$]. Then the reflection of contour C does not embrace the zero for plane f. Consequently, the increment of the argument $f(\lambda)$ equals zero and, according to (4.4), N = 0. This means that steady-state flow is stable.

2) The zero point for plane λ is reflected at a point lying on the real axis to the right of zero [$f(0) < 0$]. Then reflection of contour C embraces the zero for plane f. Since the direction of the circuit is maintained, then $\Delta_C \arg f(\lambda) = 2\pi$. According to (4.4), N = 1 means steady-state flow is unstable in this case.

In order to explain which sign of f(0) corresponds to one or another section of the HDC, it is sufficient to remember that for each tube the HDC has an N-shaped form similar to curve $a$bef in Fig. 1, whence it can be seen that for a given value of Π the slope of the HDC in absolute value in section ab is greater than in be, which in turn (with the exception of a small section to the right of point b) is greater than in ef:

$$\left.\frac{d\Pi}{d\omega_{ns}}\right|_{ab} > -\left.\frac{d\Pi}{d\omega_{ns}}\right|_{be} > \left.\frac{d\Pi}{d\omega_{ns}}\right|_{ef}.$$

Since the HDC for a system of two tubes is obtained by adding the HDC for both tubes, it is easy to establish that a condition for stability of steady-state flow f(0) > 0 is fulfilled in sections ab, bc, cg, and ef (see Fig. 1), and f(0) < 0 is fulfilled in be and eg, and flow is unstable. The boundary of stability is determined from the equality f(0) = 0, which for symmetrical flow ($v_s = 0$) coincides with the condition for origination of nonsymmetrical flows (3.4), and for nonsymmetrical flow ($v_s \neq 0$) it coincides with the condition for merging and disappearance of nonsymmetrical flows (3.5).

With $\beta \neq 0$ for symmetrical flow $v_s = 0$, $\Theta_s = \Theta_0 \exp(-2\xi/\omega)$ we have

$$f(0) = 2\int_0^1 \exp(-\Theta_s)\,d\xi - \int_0^1 \Theta_s\beta^{-1}\left[1 - \exp\left(-\frac{4\beta}{\omega}\xi\right)\right]\exp(-\Theta_s)\,d\xi.$$

Since the first integral does not depend on β, and the second decreases with an increase in β, then the region for instability of symmetrical flow contracts. In the descending branch of the HDC for symmetrical flow sections develop close to the extremes relating to stable flows (see Fig. 4). The larger these sections, the greater the β. With sufficiently large β the whole of the descending branch of the HDC becomes stable, and the effect of disturbance of symmetry disappears.

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